MEASUREMENT OF THE MOISTURE CONTENT OF ASBESTOS CEMENT BY SHF IRRADIATION

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The relation between the power loss of an electromagnetic wave passing through a freshly formed asbestos cement sheet and the moisture content of the asbestos cement has been investigated. Experimental data confirm the validity of the theoretical results obtained.

The hardening of asbestos cement sheets is largely determined by the initial moisture content of the freshly formed sheet [1]. Rapid determination of the moisture content is necessary for quality control purposes. One of the available methods consists in irradiating the sheet with SHF radiation. The moisture content can then be determined from the power loss of the electromagnetic wave in passing through the sheet.

Consider a sheet of asbestos cement of thickness d, on which a plane electromagnetic wave is normally incident. The transmission factor expressed in terms of the refractive index n (see [2], \$5) has the form

$$D = \frac{4n}{(1+n)^2 \exp(iknd) - (1-n)^2 \exp(-iknd)}.$$
 (1)

In what follows, we are concerned with the case of the complex index of refraction n, i.e., when the asbestos cement has substantial thermal losses.

In the case of complex n the transmission factor D will take into account not only the reflection of the wave from the surface of the sheet but also the internal energy absorption.

Let us compute the power transmission factor $\eta = |\mathbf{D}|^2$. For this purpose, we express (1) in the form

$$D = \frac{4n}{(n+1)^2} e^{-iknd} \left[1 + \frac{(n-1)^2}{(n+1)^2} e^{-2iknd} \right].$$
(2)

Hence we have

$$\eta = -\frac{16 (n'^2 + n''^2) \exp((-2kn''d))}{[(n'+1)^2 + n''^2]^2} \left[1 + \frac{2(n'-1)^2 + n''^2}{(n'+1)^2 + n''^2} \cos(2kn'd) e^{-2kn''d} \right].$$
(3)

Here we note that, like (2), expression (3) is valid for $\exp(-2kn^{"}d) \ll 1$. For moist asbestos cement—a material with quite strong absorption—this condition is usually satisfied.

From (3) we can obtain the wave power loss. Expressed in decibels, it is written in the form

$$A = 10 \log \eta = 4,35 \ln \eta =$$

$$= -8.7 \left[kn''d + \ln \frac{(n'+1)^2 + n''^2}{4\sqrt{n'^2 + n''^2}} - \frac{(n'-1)^2 + n''^2}{(n'+1) + n''^2} \cos (2kn'd) e^{-2kn''d} \right].$$
(4)

Let us consider the structure of this expression. The first term in the brackets gives the losses due to power absorption in the material of the sheet, the second the losses due to reflection from the surfaces, while the third expresses the interaction between the reflections from the two surfaces of the sheet. The third term is small, since it contains the small factor exp(-2kn"d).

Equation (4) gives the power loss in terms of the real and imaginary parts of the index of refraction of the asbestos cement. We express these quantities in terms of the electrical parameters and the concentrations of the components, namely: 1) asbestos; 2) cement; 3) water; and 4) air.

Freshly formed asbestos cement may be regarded as a heterogeneous mixture, since there have still not been any chemical changes. To find the index of refraction of this mixture, it is necessary to average over the electrical parameters of the components. To perform this averaging correctly, it is necessary to know the internal structure of the asbestos cement.

This structure is based on the asbestos fibers, which are for the most part parallel to the surfaces of the sheet. The cement, water, and air are more or less uniformly distributed between the asbestos fibers. To find the averaged electrical parameters of this structure it is necessary, generally speaking, to solve a very complicated electrodynamic problem, which lies outside the scope of this investigation. Therefore, we confine ourselves to a consideration of two idealized approximations of the structure in question, which, however, will enable us to define the boundaries within which the true value of the index of refraction must lie.

One of these approximations is a layered medium with the boundaries of the layers parallel to the surfaces of the sheet. For this medium [2,3]

$$\varepsilon = \sum_{m=1}^{4} q_m \varepsilon_m, \ n^{(1)} = \sqrt{\varepsilon}, \tag{5}$$

where m = 1, 2, 3, 4 according to the enumeration of the components introduced above, $\varepsilon_{\rm and} \varepsilon_{\rm m}$ are the dielectric constants of the mixture and the components, respectively, and q_m is the relative content of the components by volume.

Since, however, the asbestos fibers do not actually form layers with clearly defined boundaries, the other extreme approximation is a layered medium with absolutely no reflection from the boundaries between the layers. As is easy to show on the basis of the principle of equality of the optical paths, in this case the averaging is taken directly over the indices of refraction of the components

$$n^{(2)} = \sum_{m=1}^{4} q_m n_m, \qquad (6)$$

where n_m is the index of refraction of the corresponding component.

The true value of n will lie somewhere between these extremes: $n^{(1)} > n > n^{(2)}$. Separating the real and imaginary parts of the index of refraction in (5) and (6), we obtain:

$$n^{(1)'} = \left(\sum_{1}^{4} q_m \varepsilon'_m\right)^{1/2} \left(1 + \frac{1}{8} \operatorname{tg}^2 \delta\right),$$
$$n^{(1)''} = \frac{1}{2} n^{(1)'} \operatorname{tg} \delta, \tag{7}$$

where

$$\operatorname{tg} \delta = \frac{\sum_{1}^{4} q_{m} \varepsilon_{m}^{'} \operatorname{tg} \delta_{m}}{\sum_{1}^{4} q_{m} \varepsilon_{m}^{'}}$$

and

$$n^{(2)'} = \sum_{1}^{4} q_m n', \ n^{(2)''} = \sum_{1}^{4} q_m n_m', \tag{8}$$

where

$$n'_m = V \overline{\varepsilon'_m} \left(1 + \frac{1}{8} \operatorname{tg}^2 \delta_m \right); \ n''_m = \frac{1}{2} V \overline{\varepsilon'_m} \operatorname{tg} \delta_m.$$

Here, all the quantities have been expressed in terms of the real parts of the dielectric constants ϵ_m' and the loss tangents tg δ_m of the components, since these quantities are usually given, and we have taken into account the fact that tg $\delta_m < 1$.

Equations (7) and (8) contain the quantities q_m , but in practice it is usually the content of the components (other than air) by weight that is known. Therefore, it is necessary to express q_m in terms of quantities that express the content of asbestos, cement, and water by weight.

By definition

$$q_m = \frac{V_m}{V}, \quad \sum_{1}^{4} V_m = V,$$
 (9)

where V_m is the volume occupied by the corresponding component. This volume can be represented in the form

$$V_m = \frac{P_m}{\gamma_m}; \ m = 1, \ 2, \ 3; \ \sum_{1}^{3} P_m = P,$$
 (10)

where P_m and γ_m are the weight and density (specific weight) of the component, respectively, and P is the total weight of the specimen investigated.

From these expressions it is easy to obtain

$$\eta_m = \frac{W_m}{\gamma_m} \gamma, \tag{11}$$

where $W_m = P_m/P$ is the relative content of the components by weight $\left(\sum_{1}^{3} W_m = 1\right)$; γ is the density of the specimen as a whole. The moisture content is expressed in terms of W_3 . The following equations can be used to find γ :

$$\sum_{1}^{4} q_{m} = \gamma \sum_{1}^{3} W_{m} / \gamma_{m}^{\cdot} + q_{4} = 1,$$

$$q_{3} + q_{4} = \gamma W_{3} / \gamma_{3} + q_{4} = q_{4}^{0},$$
(12)

where q_4^0 is the content of air by volume in the completely dehydrated (dried) specimen, i.e., at $q_3 = 0$.

The second equation expresses the fact that the specimen can only fill with water at the expense of a corresponding reduction in the air content. From (12) we have

$$\gamma = \frac{1 - q_4^0}{W_1/\gamma_1 + W_2/\gamma_2} \,. \tag{13}$$

Substituting (13) in (11), we obtain

$$q_{1} = \alpha q_{2}, \ q_{2} = \frac{1 - q_{4}^{0}}{1 + \alpha},$$

$$q_{3} = \frac{\gamma_{2}}{\gamma_{3}} \frac{(1 + K) W_{3}}{1 - W_{3}} \ q_{2}, \ q_{4} = q_{4}^{0} = q_{3},$$
(14)

where we have introduced the notation $\alpha = K\gamma_2/\gamma_1$; $K = W_1/W_2$ is the relative content of asbestos by weight (in terms of the cement content), which does not depend on the moisture content.

Thus, the power loss of an electromagnetic wave passing through an asbestos cement sheet can be estimated from expressions (4), (7), (8), and (14). These expressions are rather complicated and inconvenient for practical purposes. Moreover, they tend to conceal the relation between power loss and moisture content. Fortunately, not all the factors involved have an important influence on the power losses. For calculation purposes, Eqs. (4), (7), and (8) can be somewhat simplified.

For the practical measurement of the moisture content from the power loss, it is necessary to select a range of working frequencies in which the losses in the water and, hence, $tg \delta_3$ are maximum. It is known [4] that $tg \delta_3$ has a maximum near 10 GHz or $\lambda = 3$ cm. Fortunately, standard equipment is available for this frequency band.

The temperature dependence of ε_3' and tg δ_3 in the 3-cm band has been plotted in Fig. 1 on the basis of experimental data taken from [4]. The value of ε_3' fluctuates between 40 and 60; that of tg δ_3 , between 0.2 and 1.0. At temperatures of 20-25° ε_3' has a value of 52-54, while tg δ_3 is equal to 0.6-0.55.



loss tangent (tg δ_3) of water as functions of temperature (t) at a frequency of 10 GHz: 1) ϵ_3^{+} ; 2) tg δ_3 .

The electrical parameters of asbestos and cement are as follows: $\varepsilon_1' = 1.8-2.1$, tg $\delta_1 = 0.05-0.07$; $\varepsilon_2' = 3.9-4.3$, tg $\delta_2 < 0.02$. These data were obtained at the Lebedev Physicotechnical Institute, Moscow, which, at the authors' request, measured the parameters of certain types of asbestos and cement using the method described in [5]. The range of values corresponds to the various types of asbestos and cement.

Clearly, tg $\delta_{1,2} \ll 1$, tg $\delta_{1,2} < \text{tg } \delta_3$. Keeping this in mind, together with the fact that tg $\delta_4 = 0$, $\epsilon_4 = 1$, we can introduce certain simplifications into expressions (7) and (8). The simplified expressions for $n^{(1)}$ ', $n^{(1)"}$ and $n^{(2)'}$, $n^{(2)"}$ take the form

$$n^{(1)'} = (q_1 \varepsilon'_1 + q_2 \varepsilon'_2 + q_3 \varepsilon'_3 + q_4)^{1/2} ,$$

$$n^{(1)''} = \frac{1}{2} \frac{q_3 \varepsilon_3^{-1} \lg \delta_3}{n^{(1)'}}; \qquad (15)$$

$$n^{(2)'} = q_1 \sqrt{\varepsilon_1'} + q_2 \sqrt{\varepsilon_2'} + q \sqrt{\varepsilon_3'} \times \frac{1}{2} + q_3 \sqrt{\varepsilon_1'} + q_4,$$

$$n^{(2)''} = \frac{1}{2} q_3 \sqrt{\varepsilon_3'} \lg \delta_3. \qquad (16)$$

It is also easy to show that at the above-mentioned numerical values of the electrical parameters of the components the relation $(n'')^2 \ll (n')^2$ is satisfied, which enables us to simplify the expression for the power loss (4):

$$A = -8.7 \left[2\pi n'' \frac{d}{\lambda} + \ln \frac{(n'+1)^2}{4n'} - \frac{(n'-1)^2}{(n'+1)^2} \cos \left(4\pi n' \frac{d}{\lambda} \right) e^{-4\pi n'' d/\lambda} \right].$$
(17)

Equations (14), (15), (16), and (17) were used to calculate the A vs. W_3 curves for several specimens of different composition at different sheet thicknesses and different temperatures. The results of the calculations and the experimental data for three specimens are presented in Figs. 2 and 3.

In one particular instance, the asbestos cement contained 127 kg of asbestos and 800 kg of cement. The sheet thickness d = 0.6 cm. The measurements were made at a specimen temperature of 22°. The



Fig. 2. Power loss in asbestos cement sheet as a function of moisture content at W_1/W_2 = = 0.16, d/λ = 0.19, t = 22°.



Fig. 3. Power loss in asbestos cement sheet as a function of moisture content: 1) at $W_1/W_2 = 0.16$, $d/\lambda = 0.24$, $t = 17^\circ$; 2) at $W_1/W_2 =$ = 0.15, $d/\lambda = 0.15$, $t = 25^\circ$.

density of the asbestos was $\gamma_1 = 2.5 \text{ g/cm}^3$; the density of the cement was $\gamma_2 = 3.2 \text{ g/cm}^3$; and that of the water was $\gamma_3 = 1 \text{ g/cm}^3$. The wavelength of the electromagnetic field was $\lambda = 3.2 \text{ cm}$.

From Fig. 1 we find that at 22° for water $\varepsilon_3' = 53$, tg $\delta_3 = 0.58$. In the calculations the dielectric constants of the asbestos and the cement were taken equal to $\varepsilon_1' = 2$, $\varepsilon_2' = 4$. The air content by volume of the completely dehydrated asbestos cement was taken equal to $q_4^0 = 0.55$ (see [6]). According to these data, $K = W_1/W_2 = 127/800 = 0.16$; the quantity $\alpha = K\gamma_2/\gamma_1 =$ = 0.20. The subsequent calculations were based on Eqs. (14)-(17).

It is clear from Figs. 2 and 3 that, as expected, the experimental values fall between the theoretical curves.

Thus, we arrive at the following conclusions:

1. The wave losses are composed of the thermal losses in the material itself and the losses associated with reflection from the surfaces of the sheet. At around 3 cm, the thermal losses are quantitatively determined exclusively by the losses in the water (cf. Eqs. (17), (18)) and do not depend on the losses in the asbestos and the cement.

2. Approximate analytic expressions have been obtained for the relation between wave loss and moisture content. These make it possible to determine the limits within which the actual losses lie.

NOTATION

 λ is the wavelength of the electromagnetic field; D and η are the amplitude and power transmission factors; n = n' - in'', ϵ' , δ , γ and $n_m = n'_m - in''_m$, ϵ'_m , δ_m , γ_m are the complex index of refraction, dielectric constant, loss angle, and density of the asbestos cement as a whole and of its components: m = 1, 2, 3, and 4 for asbestos, cement, water, and air, respectively; d is the thickness of the sheet; q_m and W_m are the relative content of the components by volume and weight; A are the losses in decibels.

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